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Modeling and Analysis of an Airport Departure Process

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This paper develops and implements an analytical queueing model for an aircraft departure process. The model is formulated using data collected at LaGuardia Airport in June 1994. Departure demand is represented by a nonhomogeneous Poisson process, and service times are modeled as appropriate mixtures of exponential stages. Transient analysis of the resulting Markovian system is performed to produce a time-dependent plot of anticipated departure delay. The model and analytical results yield useful insights for airport capacity estimation and departure delay prediction.

I. Introduction

GROWING congestion at major U.S. airports has created a need for better understanding of factors that can lead to aircraft departure delays. This article presents an analytical queueing model that captures key variables in the aircraft departure process and offers an approach toward airport capacity estimation and departure delay prediction. The model is developed and demonstrated using data collected from New York City's LaGuardia Airport (LGA) during a single week in June 1994. LGA was selected because the airport has characteristics common to many U.S. facilities, including the frequent occurrence of significant departure delays.

Like many airports, LGA is configured with two intersecting runways. Normal operating procedures assign a primary departure runway, which can be viewed as the single server in a queueing system. Several definitions are useful in formulating an appropriate queueing model. Departure is synonymous with service completion, which occurs when an aircraft completes takeoff and clears the runway environment sufficiently for another aircraft to be granted takeoff clearance. Service demand occurs when an aircraft enters the departure queue after leaving a passenger gate (pushback) and taxiing to the runway. Departure delay is the difference between service demand time and the initiation of service (clearance for takeoff). Roll-out time is the total time between pushback and takeoff clearance (sum of taxi time and departure delay).

Initial analysis of the LGA activity data reveals that roll-out time varies greatly with time of day, primarily because of time-dependent variation in the frequency of scheduled pushbacks. The queueing model is consequently designed to capture variability in pushback rates and runway service time to produce a time-dependent plot of expected roll-out time. While this type of transient analysis has been explored for some aspects of airport operation, the departure process has received relatively little attention. There are various published investigations of airport arrival (landing) processes.[1–6] There are also general discussions of airport capacity estimation and delay optimization.[7–10] Shumsky[11] offers a fluid dynamic model of the departure process and compares model delay predictions with actual airport observations. Shumsky's results[11] highlight the importance of accurate capacity information and suggest some value in explicitly modeling variability in component processes.

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II. Data Analysis

The data set used in this study was provided by the Federal Aviation Administration and includes inputs from the ARINC Communications Addressing and Reporting System (ACARS) and the Airport Service Quality Performance (ASQP) system. The set contains 3885 records of arrival and departure data for the eight major airlines that transited LGA from June 1–7, 1994. Each record consists of 12 fields of primary data and 27 fields of derived data. The primary fields include airline name, flight number, departure airport, Official Airline Guide (OAG) departure time, actual departure time, and aircraft type. The derived data fields include pushback delay, taxi delay, and takeoff delay. Data fields detailing weather conditions and airport operating configuration are also provided. Much of the data for June 1 and 2 are missing because of recording problems. However, sufficient data are available for the remaining days to generate useful models. The most interesting days are June 6 and 7, which experience significant weather phenomena and the most substantial delays.

To gain insight into general system behavior, a brief study of the actual delays experienced was first accomplished. Figure 1 displays a time-dependent plot of aircraft roll-out time for Monday, June 6. Roll-out time is not available from the raw data, but can be derived from other fields. Observations plotted at values of zero (26 out of 287) indicate missing data. The plots of actual roll-out times for each day clearly imply that significant nonrandom delays occur during peak periods. It is therefore apparent that the analytical model must capture the transient nature of the system.

A. Demand Process

The actual times when departing aircraft demand service are not recorded in the data set, but pushback times (with a nominal taxi time added) offer a reasonable surrogate. Plots of the number of pushbacks per hour indicate a peak demand period from 0600 to 1000 and another from 1500 to 1900. While the

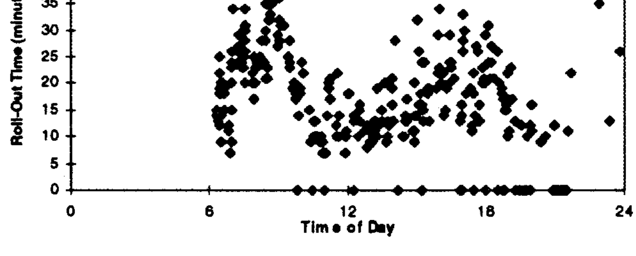

Fig. 1 Observed roll-out times.

pushback rate is clearly not homogeneous throughout the day, statistical analysis supports the hypothesis that times between pushbacks can be modeled as independent identically distributed (iid) exponential random variables within each 1-h interval. Using a 5% significance level, χ^2 and Kolmogorov–Smirnov (KS) goodness-of-fit tests (Ref. 12, pp. 519–591) both fail to reject this hypothesis for each hour. The demand process can therefore be plausibly modeled as a nonhomogeneous Poisson process (Ref. 13, pp. 31–48) with intensity function $\lambda(t) = \lambda_n$, where λ_n is the mean number (rate) of pushbacks in hour n of the day.

To obtain an estimate for nominal taxi time, it is assumed that a departure queue does not exist in lull periods where roll-out times appear stable. Average time between pushback and takeoff clearance for these observations represents approximate taxi time. By accepting the additional assumption that any delay caused by taxiway congestion is relatively insignificant, this taxi time can be used to translate all pushback times to approximate service demand times.

B. Service Process

Probabilistic modeling of the runway service time is a more difficult task and is the key to accurate modeling of the airport departure process. The service time distribution is related to the departure capacity, which is a function of the runway operating configuration and weather conditions. For example, LGA appears to operate most efficiently with arrivals on runway 22 and departures on runway 13. When the weather permits Visual Flight Rules (VFR), the airport can accommodate about 34 arrivals per hour, or roughly one every 1.76 min. When LGA is accepting arrivals on runway 13 via the Instrument Landing System (ILS), the arrival capacity is only about 20 aircraft per hour.

Note that departure service times are not synonymous with interdeparture times. The departure queue may periodically empty, and so an observed interdeparture time may include an idle period beyond the service time for the previous aircraft. External factors can also increase the time between departures. For LGA, these factors include safe clearance requirements for aircraft landing on the intersecting runway and in-flight safe separation requirements for aircraft transiting airports in the New York Terminal Control Area (TCA). As shown in Fig. 2 (as adapted from Ref. 14) these airports include John F. Kennedy International (JFK) and Newark International (EWR). It should also be noted that the LGA data set does not include comparable detail for General Aviation Aircraft (GAA), although these aircraft are also customers in the departure queue. The representations of service time used in this study attempt to account for these factors.

To obtain an initial service time representation, the times between takeoffs are analyzed for the morning and afternoon peak periods of June 6 and 7. Since all aircraft departing during these periods experience above-average roll-out times, it is assumed that the runway is never idle because of customer starvation. A histogram for observed service times during the morning peak period on June 6 is displayed in Fig. 3. Also shown are probability density functions for exponential, Erlang-2, and Erlang-3 random variables with the same mean as the empirical data. These distributions are chosen as candidate probability models for purposes of analytical tractability. The variance of the Erlang-2 distribution offers the closest match to the sample variance of the data. When χ^2 and KS goodness-of-fit tests are performed, neither the exponential nor Erlang-2 distribution is rejected at a 5% significance level.

The bimodal appearance of the service time histogram suggests that a better representation may be obtained from a probabilistic mixture of two separate conditional distributions. The first conditional distribution is postulated to represent the time between takeoffs given no external event occurs. The second distribution includes a server absence time, since the runway is presumably unavailable to aircraft in the departure queue for some time because of one or more external events (e.g., arrival or GAA departure). By visually interpreting the histogram, it is postulated that observations of 3 min or less represent samples from the first distribution (service time only) and those greater than 3 min are samples from the second

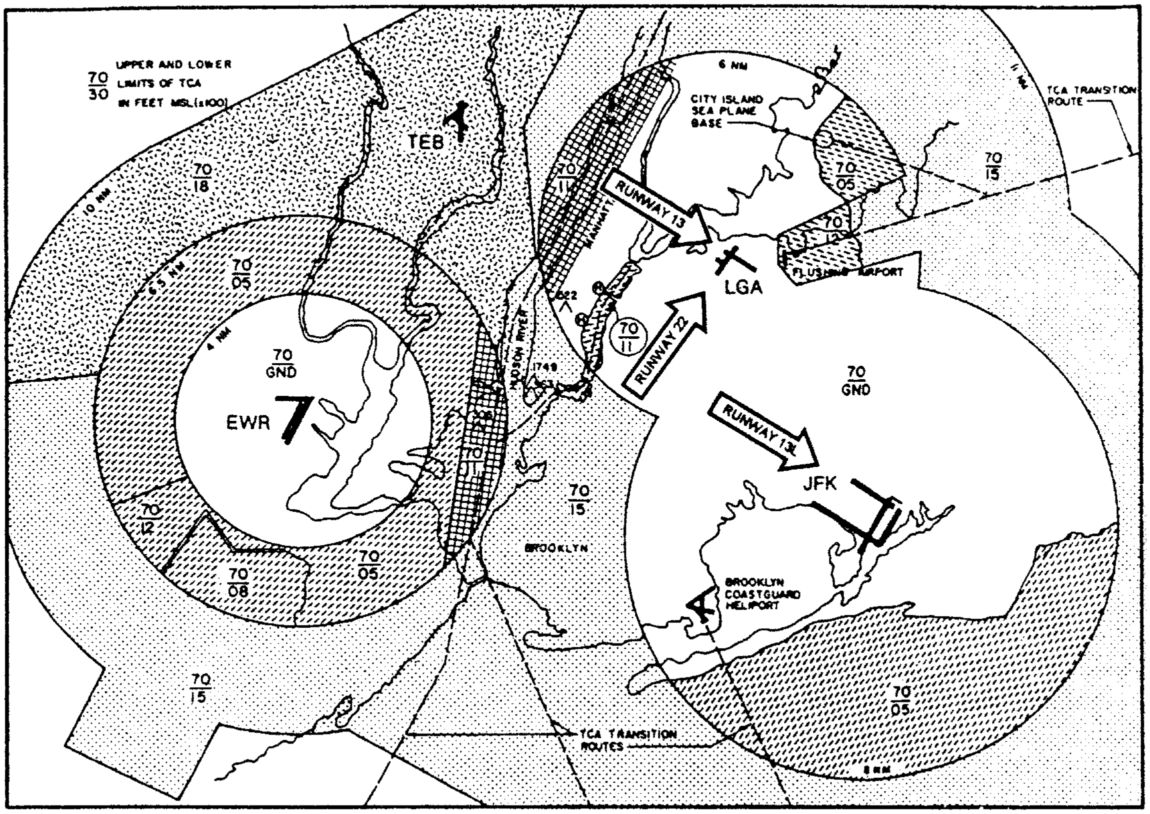

Fig. 2 New York terminal control area.

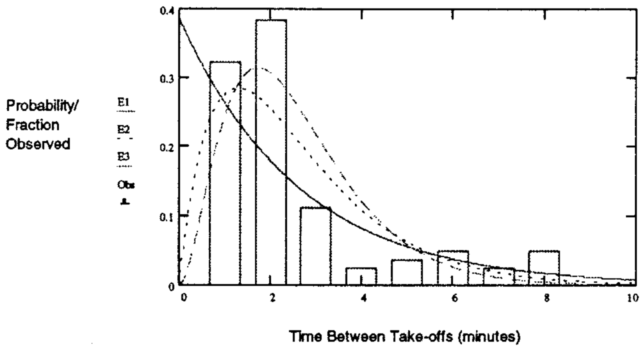

Fig. 3 Service times.

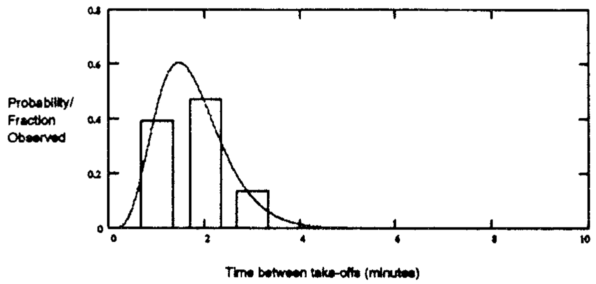

Fig. 4 Service times (excluding absences).

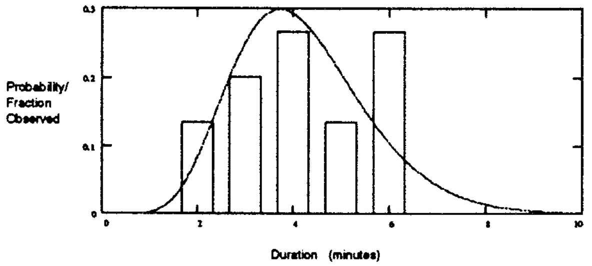

Fig. 5 Absence times.

distribution (convolution of service time and absence time). Figure 4 demonstrates that an Erlang-6 distribution is obtained when matching the first two moments of the service time observations. To estimate distribution parameters for absence time, a nominal 2-min service time is subtracted from all raw observations exceeding 3 min. As indicated in Fig. 5, the Erlang-9 distribution is chosen to represent absence time by comparable moment matching.

III. Model Development

Based on the preceding data analysis, three different queueing models are formulated and evaluated. In all cases, times between service demand are modeled as exponentially distributed random variables with demand rates that change at fixed time intervals of width δt (e.g., each hour of the day). Brief descriptions of the models follow:

1) *Exponential model*: All service times are represented by iid exponentially distributed random variables.

2) *Erlang-k model*: All service times are represented by iid Erlang-k random variables. If the mean service rate is μ, each service time can be viewed as the sum of k exponentially distributed stages with mean completion rates $k\mu$ (Ref. 15, pp. 119–130).

3) *Erlang-k model with server absences*: With probability p, a server absence is experienced between departures. Figure 6 displays a transition diagram for a notional case where service times and absence times each have two stages. Each absence stage is completed with rate 2α, so the mean absence time is α^{-1}. The actual LGA model has six service stages and nine absence stages.

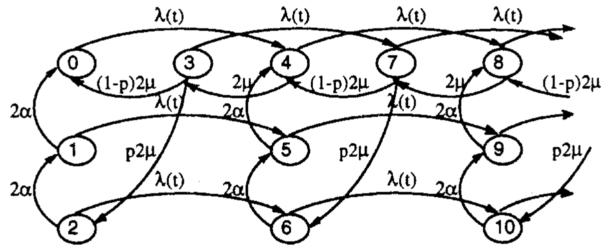

Fig. 6 Transition diagram for the absence model.

Each of these three models can be studied as a continuous-time Markov process within each homogeneous interval of airport operations. The state of the system at any time is completely described by the number of service stages remaining in the system (and the number of absence stages for model 3). The probability distribution for system state at time $t \in \{0, \delta t, 2\delta t, \ldots\}$ (where $\delta t = 1$ h) is given by the vector $\boldsymbol{P}(t) = [P_0(t), P_1(t), P_2(t), \ldots]$. Since LGA is closed daily from midnight to 0600, the initial state probability vector for $t = 0$ (0600) is a degenerate distribution representing an empty system $[P_0(0) = 1, P_i(0) = 0 \ \forall \ i > 0]$.

To determine the state probability vectors for each time interval, it is necessary to compute the transition probabilities for each possible transition from one state i at time t to another state j at time $t + \delta t$. This is accomplished by numerically solving a system of differential equations formed from state transition rates to produce a matrix of transition probabilities $P(t) = [P_{ij}(t)]$.[16] State probability vectors are computed sequentially, with the vector for each interval used as the initial condition for the subsequent interval. That is,

$$\boldsymbol{P}(\cdot)[(n + 1)\delta t] = \boldsymbol{P}(n\delta t)P(n\delta t), \qquad n \geq 0 \tag{1}$$

For computational purposes, it is necessary to truncate the state space by artificially limiting the capacity of the queueing system to an upper bound of N aircraft. For this study, an upper bound is considered reasonable if it is more than twice the magnitude of the highest average queue length generated. When this rule is employed, the state probability for the highest state is always less than 0.01, and is typically much smaller.

Once all probability vectors are determined, time-dependent performance measures can be computed. For example, expected queue length at time t for an Erlang-k model can be computed as

$$E[Q(t)] = \sum_{j=1}^{N} (j - 1) \left[\sum_{i=1}^{k} P_{k(j-1)+i}(t) \right] \tag{2}$$

When the system is in state $k(j - 1) + i$, $i > 0$, there are j aircraft in the system and $(j - 1)$ aircraft in the queue. The variance of queue length can be computed as

$$V[Q(t)] = E[Q^2(t)] - \{E[Q(t)]\}^2$$
$$= \sum_{j=1}^{N} (j - 1)^2 \left[\sum_{i=1}^{k} P_{k(j-1)+i}(t) \right] - \{E[Q(t)]\}^2 \tag{3}$$

The expected waiting time (departure delay) is given by

$$E[W(t)] = \sum_{j=1}^{N} \sum_{i=1}^{k} \frac{k(j - 1) + i}{k\mu} P_{k(j-1)+i}(t) \tag{4}$$

Since the entire probability distribution for system state is available, myriad additional performance measures can be obtained. For example, the probability that the departure queue length does not exceed a specified unacceptable value can easily be computed. Models based on fluid approximations do not yield this information.

IV. Implementation and Results

Using the LGA data set, the exponential and Erlang models are formulated and executed for all days represented by adequate data (June 3–7). The absence model is run only for June 6 and 7. Delays are most notable on these days since they experience significant weather below visual meteorological conditions (VMC). The results for June 6 and 7, therefore, form the primary basis for model evaluation. Additional results are available in Ref. 17.

To calibrate each model, the output must be correlated to the type of data recorded at the airport. While time-dependent queue length is a key model ouput, no information about actual queue length is available for comparison. However, a model estimate for expected roll-out time can be obtained by adding nominal taxi time to expected waiting time. To obtain the best model fit to the empirical data, analytical and actual roll-out times are compared at various effective service rates.

Calibration of the exponential model results in an effective average service rate of 24 aircraft/h for June 6 and 25 aircraft/h for June 7. Figure 7 displays the actual roll-out times along with the analytical roll-out times based on these rates. The figure indicates that the actual service rate may have dropped below 24 between 1500 and 1800 on June 6 and increased above 25 between 0700 and 0900 on June 7.

On Monday, June 6, LGA operated on runways 22 (arrival) and 13 (departure) for the majority of the day. While the FAA data dictionary lists this condition as the preferred configuration,[18] the airport experienced some of its most significant delays on this date. The recorded weather provides a possible explanation. The data dictionary reveals that runway 13 departures may experience delays when JFK is using a particular type of approach to runway 13L. The wind direction (between 150–180 deg) and general weather conditions support this possibility.

On Tuesday, June 7, the wind direction varied between 180–220 deg, suggesting that JFK was not employing approaches that would significantly interfere with LGA. This may explain why the effective service rate was slightly higher than on June 6, even though general weather conditions were worse. The low actual roll-out times early Tuesday morning may have been caused by relatively sparse occurrence of external events (e.g., fewer GAA departures because of poor weather).

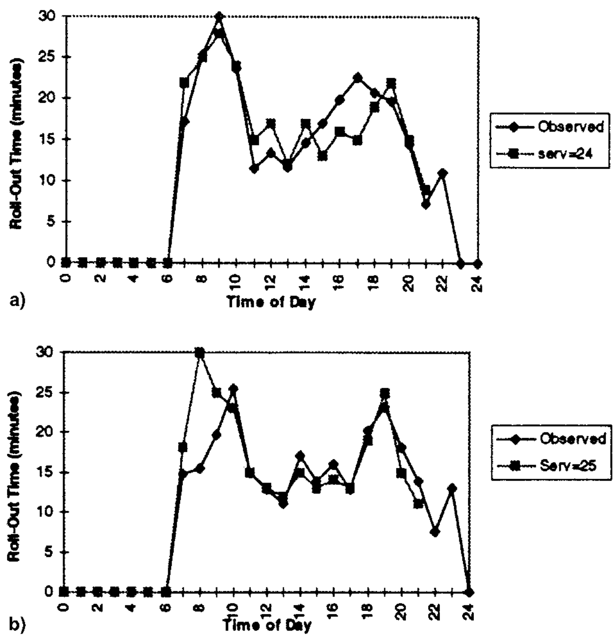

Fig. 7 Results for the exponential model: a) Monday, June 6 and b) Tuesday, June 7.

Erlang-2 model results for June 6 and 7 are displayed in Fig. 8. The respective calibrated service rates are 23 and 26 aircraft/h. When all results for June 3–7 are compared, the Erlang model appears superior to the exponential model in matching observed roll-out times. For example, the apparent overestimation of peak delay on the morning of June 7 is not as severe with the Erlang model.

Figure 9 displays results for the absence model. From the recorded data, the probability that the runway would not be immediately available to an aircraft at the head of the queue is estimated at 0.2 for June 6 and 0.23 for June 7. The service rate (when the runway is available for departures) is estimated at 35 aircraft/h for both days. While the absence model would intuitively appear more realistic than the Erlang model, comprehensive review of the model results does not indicate substantially better performance. This comparability is typified in Figs. 8 and 9. Note that the implementation used in this study did not attempt to isolate hourly absence probabilities. This

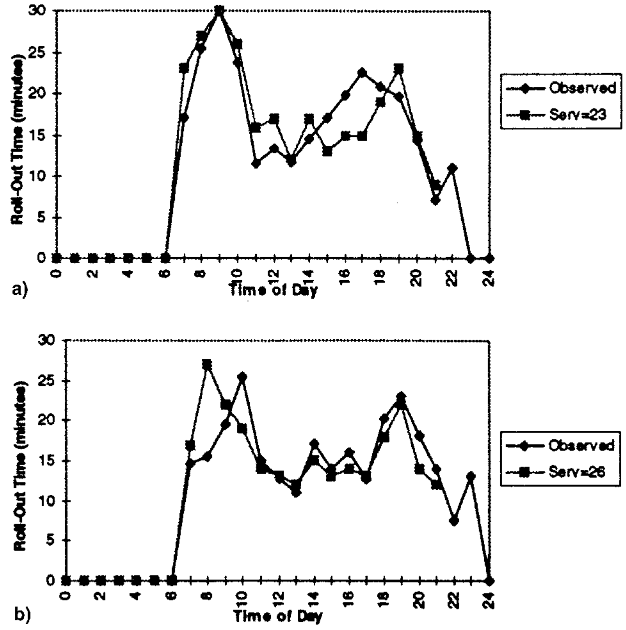

Fig. 8 Results for the Erlang-2 model: a) Monday, June 6 and b) Tuesday, June 7.

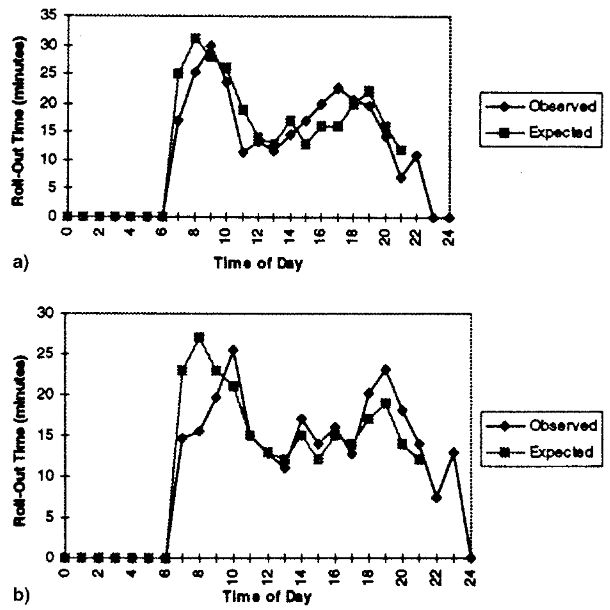

Fig. 9 Results for the absence model: a) Monday, June 6 and b) Tuesday, June 7.

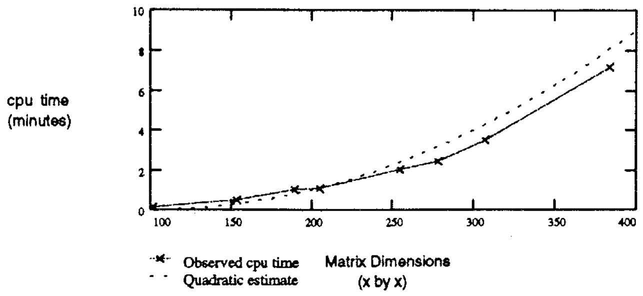

Fig. 10 Computational performance.

extension could potentially improve the performance of the absence model.

All three models are implemented in Fortran and executed on a UNIX, SPARC-20 computer system. IMSL subroutines are employed for the matrix operations required to determine the transition matrix $P(t)$. As indicated in Fig. 10, computation times are quadratically related to the number of states in the model. The Erlang-2 model (with a maximum queue length of 25) has 52 states. About 20 s of CPU time are consumed in processing 15 time intervals. As suggested in Fig. 10, computation times are significantly affected by the increased dimensionality of a typical absence model. The absence model employed in this study has 385 states, requiring 7.5 CPU minutes for one time interval and approximately 2 h to complete all 15 intervals.

V. Conclusions

Three models for analysis of an airport departure process are developed and evaluated in this study. The Erlang model appears to offer the best overall performance in terms of simplicity, accuracy, and computational efficiency. Results produced through this model correlate well with the actual rollout times observed at LGA. If the LGA data set reflected all scheduled departures (including GAA), the service time distribution would likely be more symmetric and have a lower variance. This distribution could be captured by increasing the number of stages in the Erlang model. Such flexibility suggests that the model could also be applied to airports that differ significantly from LGA. For example, under a situation where arrivals and departures employ the same runway, periodic preemption of the runway by blocks of landing aircraft could be captured in the service time distribution. Current models do not provide this kind of capability. Another advantage of the new model is that airport departure capacity can be determined based on desired performance standards (e.g., limits on expected delay or probability of excessive delay). Currently, the usual approach is to estimate departure capacity as simply the service rate observed during peak periods.[10]

Each of the offered models could be used to improve the estimation of effective departure capacity at major airports. The correlation of this capacity with factors such as GAA departures, weather, and operating configuration would also be helpful in predicting queue lengths and departure delays. Whereas the determination of effective service rates is accomplished using actual pushback times, real-time delay prediction could be performed using OAG scheduled pushback times. If effective service rates were not known for a given set of conditions, the service rate from a preceding time interval could be used in estimating delays during a subsequent interval.

At the present time, departure delay prediction is virtually nonexistent. A delay is only identified if an aircraft has not departed within 5 min of its predicted takeoff time (OAG pushback time plus taxi time). Predicted takeoff time is incremented in 5-min intervals until the aircraft departs or an hour has passed. The aircraft is then deleted from the system. The models developed in this study may be helpful in improving this process.

The models may also be useful for strategic planning and scheduling. Using the June 6 pushback data and an effective service rate of 23 aircraft/h, the Erlang model yields a 0.10 probability that the queue length will exceed 17 aircraft at 0900. This potentially ugly queue suggests that an adjustment to the departure schedule may be desirable. By opening the airport 1 h earlier and dispersing scheduled pushbacks, the queue length under the same conditions would be less than 11 aircraft with probability 0.90. Mean roll-out time would remain below 22 min throughout the day.

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